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Exploration 24 sinusoidal equations from graphs answers

Prekalculus, Graph, Survey, Source, Sketch, Equation, Trigonometry, Sampler, Vertical, Curriculum, Math.kendallhunt.com Chapter 3 Resources (PDF) Transformation Sinus and Cosine Chart – Survey by Sharon K. O'Kelley This is a survey for advanced algebra or Precalculus teachers who have introduced their students to basic sinus and cosine charts and now want their students to explore how changes in equations affect graphs. This is an introductory lesson aimed at connecting the language of algebraic transformations with the more advanced theme of trignonometry. (The key follows the end of the survey.) 1. Consider the basic sinus equation and graph. Let's call it the first feature.... 2. If the first function is overridden as.... then the values a = 1, b = 1 and c = 0. Let's find out what happens when these values change.... 3. Look at the blue and red chart and their equations. The chart of the first function remains in black. Equation of the blue chart Equation of the red chart a. Describe how the equation of the first function changed and became a blue and red chart equation. This value is called the chart amplitude. B. Describe how the chart of the first function has changed to become a blue chart and a red chart. Be as precise as possible. Include in the response how a specific point in the chart of the first function transforms to become a point on the blue chart and in the red chart. c. Look at the animation of this phenomenon. Click here.... (Be patient. These movies take a while to load ....) 4. Now consider the purple and green chart and their equations. The chart of the first function remains in black. Equation of the purple chart Equation of the green graph a. Describe how the equation of the first function changed and became a purple and green chart equation. B. Describe how the chart of the first function has changed to become a purple chart and a green chart. Be as precise as possible. Include in the response how a specific point in the chart of the first function transforms to become a point on the purple chart and in the green chart. c. How does b work differently from and in the equation? Look deeper than horizontal versus vertical.... d. Look at this phenomenon .... Click here.... 5. Explore the purple and green chart and their equations. The chart of the first function remains in black. Equation of the red chart Equation of the green chart a. Describe how the equation of the first function changed and became a purple and green chart equation. B. Describe how the chart of the first function has changed to become a red chart and a green chart. Be as precise as possible. Include in the response how a specific point in the chart of the first function transforms to become a point on the purple chart and in the green chart. c. See how it moves .... Click here.... 6. Consider the blue and purple chart and their equations. Chart of the first remains in black. This value is called a phase shift chart. Equation of the blue chart Equation of purple chart a. Describe how the equation of the first function changed and became a chart equation in blue and purple. B. Describe how the chart of the first function changed to become a blue chart and a purple chart. Be as precise as possible. Include in your response how a specific point in the chart of the first function transforms to become a point on the blue chart and purple chart. c. How does c work differently when it is inside or outside parentheses? Look deeper than horizontal versus vertical.... d. Take a look.... Click here.... 7. What about the negatives? A. See the following charts.... (The first function is black.) Equations of the red chart Equation of blue chart b. Compare the charts. Describe in detail the transformations you see. 8. Throw it all together.... Consider the chart .... (The first function is black.) Describe the transformation fully. Be as precise as possible. (Tip: Look at this problem as .) 9. Kosinus chart a. On a sheet of millimeter paper, predict what the following charts will look like. 10th Let's go a little further.... Write two different equations for the same chart below. Use a sinus in one and a cosine in the other. Use graphics software or a graphics calculator to verify your response. 11. Summarize what you have learned here. Use definitions and examples. Be specific. The key to the survey 3. A. Value a is 2 a . B. The blue graph is stretched vertically by a coefficient of 2. This can be determined by comparing a point (1.57, 1) to the first function with a point (1.57, 2) on a blue chart. The red chart shrinks vertically by the factor . This can be determined by comparing the point (1.57, 1) to the first function with the point (1.57,) on the red chart. 4. a. The value b is 2 a . B. The purple chart shrinks horizontally by a factor . This can be determined by comparing the point (1.57, 1) to the first function with the point (0.785, 1) on the purple chart. The green chart is stretched horizontally by a factor of 2. This can be determined by comparing the point (1.57, 1) to the first function with the point (3.14.1) on the green chart. c. The values for a and b appear to be reversed. In other words, when and it's vertical shrink . However, if b , the horizontal section is a coefficient of 2. This can easily be explained by the fact that you are dealing with, so to be sure, for a given variable. For example, it is equivalent ; therefore, when the variable is isolated, it appears to be straightforward. However, x is not isolated in these cases.... 5. a. The value c is 2 a . B. The red chart is translated by 2 units. This can be determined by comparing the point (0.0) on the first function with the point (0,2) on the red chart. The green chart is translated by 2 units. This can be determined by comparing the point (0, 0) on the first function with the point (0,) on the green chart.) 6. a. the value c is 2 a . B. The blue chart is translated to the left of 2 units. This can be determined by comparing the point (0.0) to the first function of the point on the blue chart. The purple chart is translated to the right of 2 units. This can be determined by comparing the point (0,0) on the first function with the point (2.0) on the purple chart. c. Outside parentheses, positive c is up and negative c is down. Inside, it seems the opposite of what you'd expect. Positive c is left and negative c is right. 7. They look the same! Why? Because the sinus is a periodic function, vertical reflection and horizontal reflection provide the same image. 8. The chart translated the correct units. It was scaled down by a factor and shrunk vertically by a factor . 9. * (red) * (blue) * (purple) * (green) * (yellow) 10. Possible answer: and 9/11. Students' responses should include definitions and descriptions of transformations, as well as how the general equation changes. They should also define the amplitute and phase shift. Return Transcription Title: Group Members: Survey 3-1a: Periodic Daytime Temperatures Date: Goal: Transform the cosine function to match, approximately, the average daytime temperature data for the city. Here are the average daily high temperatures for San Antonio, by month, based on data collected over the past 100 years and published by NOAA, the National Oceanic and Atmospheric Administration. This data is used, for example, in the design of heating and air conditioning systems. Monthly temperature (−F) Monthly temperature (−F) January. 61.7 July 94.9 February 66.3 August 94.6 March 73.7 September 89.3 April 80.3 October. 81.5 May 85.6 November 70.7 June 91.8 December 64.6 4. The temperature chart in problem 1 has a high point at x = 7 months. What transformation would you apply to the sinusoids in problem 2 (dashed in the following illustration) to have a high point on θ = 7− (solid) instead of θ = 0−? Type the equation and confirm it by plotting it on the grapher. y 1 1. On graph paper, plot average daily high temperatures for two years. Suppose January is month 1 and so on. Specify a time-efficient way for group members to render. What should you render in a month zero? Tie the points with a smooth curve. y θ 12° 1 24° 7° 5. The average of the highest and lowest temperatures in the table is 94.9+ 61.7 = 78.3. Type an equation for the transformation that would translate the graph in issue 4 up by 78.3 units. 100 90 Temperature (°F) 80 70 60 50 40 30 20 10 x 6 12 Months 18 24 2. Graph y = cos θ completes the cycle every 360− (angle, not temperature). What horizontal expansion factor would be the complete cycle of each 12-, as shown? Write an equation for this transformed sinusoid and plot it on the grapher. y θ 12° 24° 6. 94.9 the highest point in problem 1 is 16.6 units above 78.3 and the 61.7 low point is 16.6 units below 78.3. Write an equation for transformation that would dilate the sinusoid in the problem 16.6 to make it look like this chart. Confirm your answer with a grapher. 94.9 78.3 61.7 θ 7° 7. On the grapher, plot the points that you plotted in issue 1. How well does the sinus equation in problem 6 fit the points? 1 3. The Earth rotates 360 times around the sun in 12 months. How do these numbers relate to the expansion factor you used in issue 2? 52 / Masters Survey 8. What have you learned as a result of doing this survey that you didn't know before? Precalculus with Trigonometry: Instructor's Source Book, Volume 1 ©2003 Key Curriculum Press Name: Group Members: Survey 3-1b: Sine and Cosine Charts, Manual Date: Target: Find the shape of sine and cosine graphs rendering them on millimeter paper. 1. On the grapher, create a table of values y = sin θ for every 10− from 0− to 90−. Set the mode to round to 2 decimal places. Plot the values on this millimeter paper. Also fence y = sin θ for every 90- to 720−. Tie the points to a smooth curve and follow the shape you have drawn for 0− to 90−. y 1 90° 180° 270° 360° 450° 540° 630° θ 720° 540° 630° θ 720° 1 2. Plot the graph y = cos θ scored, just as you did for the sinus in problem 1. y 1 90° 180° 270° 360° 450° 1 3. Find sin 45- and cos 65−. Show that the corresponding points are in issues 1 and 2. 5 in the charts. What are the ranges of sinus and cosine functions? 6. Name a real-world situation where variables are related to a periodic chart, such as a sinus or cosine. 4. Find inverse trigonometric functions θ = sinD1 0.4 and θ = cosD1 0.8 Show that the corresponding points are in the graphs in issues 1 and 2. Precalculus with Trigonometry: Resource Book Instructor, Volume 1 ©2003 Key Curriculum Press 7. What have you learned as a result of doing this survey that you didn't know before? Survey Masters / 53 Title: Group Members: Survey 3-2a: Transformed Sinusoid Charts Date: Goal: Due to the equation of the transformed sinusoid, sketch chart, and vice versa. 1. Write a horizontal expansion factor, dot, amplitute, phase displacement and vertical displacement, and sketch the chart. 3. Once you know the connection between the sinusoid equation and its graph, you can go back and write an equation from that chart. For the following sinusoids, type a dot, horizontal expansion factor, amplitum, phase displacement (for cosine function) and vertical displacement. Then write a specific equation. y = 4 + 3 cos 2(θ D 70−) Horizontal dilation factor: Period: Amplitude: Period: Phase displacement: Horizontal expansion factor: Vertical displacement: Amplitude: Phase displacement: y Vertical displacement: Equation: y θ θ 10° 2. Type the horizontal dilation factor, dot, amplitute, phase displacement, and vertical displacement, and sketch the chart. 8 y = D2 + 4 sins 30(θ + 1−) 4. Confirm that your answer to issue 3 is correct by entering the equation in the grapher and plotting the chart. Has the chart matches the number? Horizontal expansion factor: Period: Amplitude: Phase displacement: 5. According to the most time-effective method possible, find y for your equation in problem 3 if θ = 35−. Write an answer to as many decimal places as the grapher will give you. Draw something on a given chart to show that your answer is reasonable. Vertical displacement: y θ 6. What have you learned as a result of doing this survey that you didn't know before? 54 / Survey Masters Precalculus with Trigonometry: Instructor's Source Books, Volume 1 ©2003 Key Curriculum Press Name: Group Members: Survey 3-2b: Sinus equations from Graphs Date: Goal: Due to the sinusoid sketch equation, and vice versa. 1. Sketch two cycles of this sinusoid: 5. This is a half-cycle of sinusoids. Write an equation. y = D3 + 5 h sin 4(θ D 20−) y 4 θ θ 400° 600° 3 2. Write a specific equation (cos) for this sinusoid: 6. This is a quarter cycle of sinusoids. Write an equation. y 100 y θ 24° 10 3° θ 36° 60 12° 3. Write a specific equation for sinusoids in problem 2 using the sinus. 4. Plot the equation in issue 2 as y1 on the grapher. Plot the equation in issue 3 as y2. Use a different style for each chart. Do both charts match the chart? 100 7. On the sinusoid in problem 2, mark the point of inflection. Mark the next point at which the chart increases but concaving down. 8. What have you learned as a result of doing this survey that you didn't know before? Precalculus with Trigonometry: Instructor's Source Book, Volume 1 ©2003 Key Curriculum Press Survey Masters/55 Title: Group Members: Survey 3-3a: Tangent and Secant Charts Date: Goal: Discover what tangential and secant function charts look like and how they relate to sinus and cosine. No charts allowed for issues 1-7. 4. Based on the quotient property, determine where the θ-intercepts for the graph are = tan θ. Mark these captures in the image in issue 3.1. The reciprocal property states that sec θ = 1 cos θ Without grapher, use this property to sketch graph y = s θ on the same axis as the parent function graph y = cos θ. In particular, see what happens to the chart seed everywhere cos θ = 0. 5. At θ = 45−, the sin θ and cos θ are the same. Based on this fact, what equals tan 45−? Mark this point in the chart in issue 3. Mark all other points where | sin θ | = | cos θ |. tan 45− H 6. Using the points and asymptote you have marked, sketch the chart y = tan θ in the figure in issue 3. (Grafers enabled!) 7, 2015, in New Check your charts with your instructor. Graphers enabled for remaining issues. 1 θ 90° 90° 180° 270° 360° 450° 540° 8. On the grapher, plot the graph y = csc θ. Sketch the result here. 9. On the grapher, plot the graph y = children's postt θ. Sketch the result here. 2. Write a quotient property expressing tan θ as a proportion of two other trigonometric functions. 10. At what values θ are inflection points for y = tan θ? Explain why the tangential function has no critical points. 3. The following illustration shows the parent functions y = sin θ and y = cos θ. Based on the answer to issue 2, determine where the asymptotes for the y = tan θ chart are and mark them in the figure. 11, 2015, in New York. Explain why the graph y = s θ has no inflection points, even if the graph goes from concaving to concaving down in different places. 1 θ 90° 90° 180° 270° 360° 450° 540° 12. What have you learned as a result of doing this survey that you didn't know before? 56 / Survey Of Precalculus Masters with Trigonometry: Instructor's Source Books, Volume 1 ©2003 Key Curriculum Press Name: Group Members: Survey 3-3b: Transformed Tangent and Secant Charts Date: Goal: Sketch Transformed Tangent, Cortangent, Secant, and Cocooned Charts, and Find Equations From Given Graphs. 1. For y = 3 + 12 tan 5(θ D 7−), status 5. For y = 1 + 3 csc 4(θ + 10−), indicate Horizontal dilation: Horizontal dilation: Period: Period: Horizontal translation: Horizontal translation: Vertical dilation: Vertical dilation: Vertical dilation: Vertical dilation: Vertical dilation: Vertical dilation: Vertical value: 2. Sketch y = 3 + tan 5 (θ D 7−), showing vertical asymptotes, horizontal axis, inflection points, and other significant points. 1 2 6. Sketch chart y = 1 + 3 csc 4(θ + 10−), showing vertical asymptotes, horizontal axis and critical points. y θ θ 3. For the next status chart 7. For the next graph, indicate Horizontal dilation: Horizontal dilation: Period: Period: Horizontal translation (for cortangent): Horizontal translation (for secan): Vertical dilation: Vertical dilation: Vertical displacement: Vertical displacement: y 7 9° 6° 21° 36° 51° θ 1 4. Write a specific equation for the chart in issue 3. Check your response by plotting on the grapher. 4 1 20° θ 70° 160° 8. Write a specific equation for the chart in issue 7. Check your response by plotting on the grapher. 9. What have you learned as a result of doing this survey that you didn't know before? Precalculus with Trigonometry: Instructor Resource Book, Volume 1 ©2003 Key Curriculum Press Survey Masters/ 57 Title: Group Members: Survey 3-4a: Radian Angle Measures Date: Goal: Discover how radian angles are measured by measuring around a circle with a flexible ruler. in the case of 1. The figure shows a circle centered at the beginning of the system with uv coordinates. On the elastic ruler (this will be done by the index file), mark the length equal to the radius of the circle. Start at the point where the circle intersects the positive u-line and bend the ruler to mark arcs of 1, 2 and 3 units counterclockwise around the circle. 2. Draw a beam from the beginning with a point corresponding to the length of 1 radius. The drawing should look like this: 4. From the geometry, remember that the circumference of the circle is 2πr, where there is a radius. So there are 2π in a complete revolution. The fact that there are 360 – in a complete revolution gives you a way to transform between degrees and radians. Calculate exactly the number of degrees in 1 radian. How is the measured value in issue 3 compared to this exact answer? in 2 1 1 radius length 1 radian 3 u 5. Calculate the exact number of degrees in 3 radians. Draw a 3-radian angle on the previous image. How close is the degree of your drawn angle to the exact value? 3. The resulting angle in the standard position has a dimension of 1 radian. Measure the number of degrees at this angle. 6. What have you learned as a result of this survey that you did not know before? 58/ Survey Masters Precalculus with Trigonometry: Instructor's Source Books, Volume 1 ©2003 Key Curriculum Press Name: Group Members: Survey 3-4b: Radian Measures Of Angles Date: Goal: Discover How Angles in Radians Are Measured by Wrapping a String Around a Circle. 1. On the board, plot the horizontal and vertical u and v axis. Get the roll of the masking tape and place it with the center at the beginning. Draw a circle on the board by following around the outside of the roll. v Draw this circle. 7. You should answer 2π radians for problem 6. The fact that there are 360 − in a complete revolution gives you a way to transform titles into radians, and vice versa. Calculate exactly the number of degrees in 1 radian. How is the measured value in problem 5 compared to this exact answer? for Tape Roll 2. Remove the roll of tape from the board. Mark the ruler on a piece of string with units equal to the radius of the circle you drew. Then attach the string to the roll of the tape. 8. Calculate the exact number of degrees in 3 radians. On the drawing of the board, show the angle of 3 radians. How close is the degree of your drawn angle to the exact value? 3. Put the roll back on the plate so that the starting point on the string is on the positive side of the u-side of the wasps. Wrap the chain counterclockwise around the roll tape. Make marks on the board in points 1, 2, 3, 4, 5 and 6 on the string. Then remove the tape again. 4. Draw the spokes through the points that you marked on the board, such as this: in 1 2 1 length radius 1 radian 3 9. Explain why the radian size would be the same regardless of the size of the circular object you use instead of the tape roll in issue 1. u 6 4 5 5. The mean angles formed by the spokes you drew have dimensions of 1, 2, 3 , . . Radians. Measure using a prochometer to determine approximately how many degrees there are in 1 radias. 10. What have you learned as a result of this survey that you did not know before? 6. Angle 6 radians is not a complete revolution. How many radians would it take to create a complete revolution? Enter the exact value. Precalculus with Trigonometry: Instructor Book Source, Volume 1 ©2003 Key Curriculum Press Survey Masters/ 59 Title: Group Members: Survey 3-5a: Cyclic Function Parent Date: Target: Plot circular functional sinusoids and dot charts. 1. Sketch the parent trigonometric function y = sin θ. y 1 6. With the grapher still in radial mode, plot the graph of the circular function y = tan x. Sketch the result. y θ 360° 720° 1 1 2. Sketch the parent trigonometric function y = cos θ. 1 y x π 2π 3π 4π 1 θ 360° 720° 1 3. Sketch the parent trigonometric function y = tan θ. y 1 7. The only difference between the parent charts for the circular sinusoid function and the common trigonometric function of the sinusoid is the period. Explain how the periods of these two types of sinusoids relate to degrees and radians. θ 360° 1 720° 4. Set the grapher to radial mode. Set the window with the range x [0, 4π] and the y-range, as shown in the given graphs. Then plot the graph of the circular function y = sin x. Sketch the result. 8. The graph here is a transformed circular function of the sinusoid. Using what you have learned about transformations, find specific equations of this sinusoid. Confirm with grafer that your equation is correct. y y 1 x π 2π 3π 4π 1 1 5. With the grapher still in radial mode, plot the graph of the circular function y = cos x. Sketch the result. y x 10 9. What have you learned as a result of doing this survey that you didn't know before? 1 x π 2π 3π 4π 1 60 / Masters Precalculus Survey with Trigonometry: Resource Book Instructor, Volume 1 ©2003 Key Curriculum Press Name: Group Members: Survey 3-6a: Sinusoids, Since y, Find x Numerically Date: Target: Find specific equations for a given sinusoid and use it graphically and numerically to find x-values for a given value y. y 2 x 5 5 10 15 1. For the sinusoid shown, draw a line y = 5. Read the six x values from the chart for which the line interseachs the displayed part of the chart. Write your answers to one decimal place. Xh. 20 25 7. Put the field in the image at a point whose x value is not the answer to problem 5 or 6. They use an element to intersect one of these values x. xH 2. Write an equation for this sinusoid. 8. Add period multiples to x values in issues 5 or 7 to find the other two x values that are also in the chart. Tell me what number of periods you've added. 3. Plot the equation from problem 2 on the grapher. Does it look like a given chart? Multiple, n H: xH Multiple, n H : xH 4. Watch the graph in problem 3 through x = 17. Does your chart have a high point there? 5. Mark with the left point in the given graph, at which y = 5. Plot line y = 5 and use the element to intersect to find the x value at this point. 9. By adding a corresponding multiple of the period to respond to issue 5 or 7, find the first x value greater than 1000 for which y = 5. At this x value, will y be increased or decreased? How do you know? xH Multiple, n H 6. Additional x values for which y =5 can be found by adding period multiples to the x value in issue 5. Let n be the number of periods that you add. Find two x values for which y = 5. Circle three x values in issue 1 that are also the answer to problem 5 and this problem. More, n H 1: xH Multiple, n H 2: xH Precalculus with Trigonometry: Instructor Resource Book, Volume 1 ©2003 Key Curriculum Press : xH 10. What have you learned as a result of doing this survey that you didn't know before? Survey Masters / 61 Name: Group members: Survey 3-6b: Since y, Find x Algebraically Date: Target: Because of the specific equation for the sinusoid and the value of y, calculate the corresponding values x algebraically. y 16 2 x 5 5 10 15 1. Sinusoid has equation y = 9 + 7 cos 2π (x D 4) 13 20 25 4. Circle points in a given chart where line y = 5 splits the chart. For each point, communicate the value of n at that point. Confirm that this equation provides the correct y value at x = 15. 5. Find two x values if n = 100. 2. Your goal is to find algebraically the values x given y = 5. Replace 5 for y. Then the algebra necessary to get x using arccosine. Type the generic solution in the form x H (number) C (dot)n or (number) C (dot) n 6. Find the first x value greater than 1000 for which y = 5. What n equals there? 3. Write two x values from the general solution in the n = 0 row of this table. Add and subtract period multiples to fill in the other rows in the table with additional possible x. n x1 7 values. What have you learned as a result of doing this survey that you didn't know before? x2 D1 0 1 2 62 / Survey Masters Precalculus with Trigonometry: Instructor Resource Book, Volume 1 ©2003 Key Curriculum Press Name: Group Members: Survey 3-7a: Chemotherapy Problem Date: Goal: Use sinusoids to predict events in the real world. Chemotherapy Problem: Ima patient has cancer. He has to go to chemo every three weeks. One of the side effects is that her red blood cell count decreases and then returns between treatments. He will receive treatment on January 13 (the 13th day of the year). At that time, her red blood cell count is at a maximum of 800. Halfway between treatment, the number is reduced to a minimum of 200. Suppose the number of red blood cells varies sinusoidally with the day of the year, x. y (number of red blood cells) 1000 x (days) 10 20 1. Draw a sinusoid chart on the given wasle. Displays enough cycles to fill millimeter paper. 2. Write a specific equation for the (circular) sinusoid in issue 1. It is recommended to use the cosine function. 3. Enter the equation in the grapher. Plot the chart using the window that appears. Explain how the chart verifys the correctness of the equation. 30 40 50 5. View on the graph the interval of data between which Ima will feel good when it returns from a low point after treatment January 13th. 6. Find exactly the x values at the beginning and end of the interval in problem 5 by setting y = 700 and using appropriate numeric or graphical methods. Describe what xH and x H 4. Ima feels good if the red blood cell count is 700 or more, bad if the count is 300 or less, and so-so if the number is between 300 and 700. How will he feel on his birthday, March 19th? Explain how you came to your answer. 7. What have you learned as a result of doing this survey that you didn't know before? Precalculus with Trigonometry: Instructor's Source Books, Volume 1 ©2003 Key Curriculum Press Survey Masters/63 Title: Group Members: Survey 3-7b: Oil Well Problem Date: Target: Use sinusoids to predict real-world events. The figure shows a vertical cross-section of a piece of earth. The y-axis is pulled from the ground at a fence bordering the land owned by your boss, Earl Wells. Earl owns land to the left of the fence and is interested in buying land on the other side to drill a new oil well. Geologists have found an oil formation beneath Earl's earth that they believe is sine-shaped. At x = D100 feet, the upper surface of the formation is the deepest, y = D2500 feet. A quarter cycle closer to the fence, at x = D65 feet, the upper surface is only 2,000 feet deep. The first 700 feet of land behind the fence is inaccessible. Earl wants to drill in the first suitable place for x = 700 ft. y Fence 100 65 30 Inaccessible land Available land x x = 700 ft y = 2000 ft y = 2500 ft Upper surface 1. Find a specific equation for y as function x. 2. Plot the chart on the grapher. Use a window with a range of x [D100, 900]. Describe how the chart confirms that your equation is correct. 3. Find graphically the first interval of x values in the available soil, for which the upper surface of the formation is no more than 1600 feet deep. Draw a sketch that shows what you've done. 64 / Masters Survey 4. Find algebraically the x values at the ends of the interval in issue 3.5. Suppose that the original measurements were slightly inaccurate and that the x value shown on D65 feet was instead at x = D64. Would this fact make a big difference in answering problem 3? Use a time-efficient method to get an answer. Explain to me what you did. 6. What have you learned as a result of this survey that you did not know before? Precalculus with Trigonometry: Instructor Resource Book, Volume 1 ©2003 Key Curriculum Press Name: Group Members: Survey 3-8a: Sinusoid Test Test Date: Goal: Use circular sinusoid functions as mathematical models. Degrees and Radians Problems: The image shows a unit circle in the uv-coordinate system. The x-axis of the xy-coordinate system is located dotily to the circle with its origin, x = 0, at the point (u, v) = (1, 0). In X Archaeology Problem: The picture shows part of an ancient wall. Archaeologists assume that the height of the wall was a sine function of the distance from the left end of the wall, with a low point y = 3 m at x = 0 m and another high point y = 7 m at x = 4 m. Rest the wall fell apart. However, the end of the wall on x = 27 m is clearly visible. Problems with response 7-10. y 5 1 3 x u 1 1. Suppose the x-axis is around the unit circle. Sketch the points at which x = 3, 2, 1, and D1 are mapped to the unit circle. 5 10 15 20 25 7. Mark all high points and all low points that the wall had between x = 0 and x = 27, inclusive. Then sketch the chart. 8. Write a specific equation for y as a function x. 2. Show the angle of 3 radians in the standard position in the figure. 3. The angle of radian D1 shall be displayed in the standard position in the figure. 4. Calculate the number of degrees in 3 radians. Type the answer in decimal form with three dots. 5. Calculate the number of radians in 240−. Simplify the answer, but leave it accurate (without decimal places). 6. If you draw a circle with a radius of 10 cm and then measure the 7 cm arc with an elastic ruler, what would be the radian rate of the central angle that covered the arc? 9. Based on your mathematical model, what equals the right end of the wall? 10. Find numerically the first value x above 10 m, at which the wall height was 3.6 m. (Over) Precalculus with trigonometry: Instructor's Source Books, Volume 1 ©2003 Key Curriculum Press Survey Masters / 65 Title: Group Members: Survey 3-8a: Test for Sinusoids Test Tide Problem: (average) water depth at a certain point on the beach moves sinusoidally with time due to the movement of tides. The figure shows the depth, y, measured in feet, at such a point as function x, measured in the hours after midnight on early January 1. The specific equation of sinusoids is y = 3 + 4 cos continuation Date: 14. At the beginning of World No 2, the European Is it peak on January 1st or January 2nd? Displays calculations that curve your response. π (x D 1) 5.8 y 15. Find graphically the first interval of times 1. 5 x 12 24 11. What is the deepest water it gets? Since the first time on 1 January 2004, the Commission has been What is the period of this feature? 16. Calculate algebraically for the first time on January 3 (i.e. x ≥ 48), at which the water depth is exactly zero. 12. If the graph drops below the x-axis, the water is completely gone, leaving a point on the beach from the water. At what age does the lowest tide first occur on January 1? How deep would you have to dig the sand for water to flow into it at the time? 17. What have you learned as a result of this survey that you did not know before? 13. Calculate the water depth at 16:00 p.m. Show that the answer matches the chart. 66 / Survey Masters Precalculus with trigonometry: Instructor is the source of the book, Volume 1 ©2003 Key Curriculum Press 6. y H 78.3 C 16.6 cos 30(θ D 7−). In fact, it should be y H 78.3 C 16.6 cos 30(t D 7), where t is the time in months. 2-5c 1. Follow the instructions in the text to draw. y 2. x M 580 m, y M 450 m 3. tan 27− = 50 y y , tan 38− = 307 + x θ 10 + x 4. Rewriting equations as cot 27- = 307 m = 307 m + yx y 20 7. The save is displayed only for the first year. The second year is the same. Fit is good but not perfect. and cot 38− = yx , you get 307 m y = cot 27− D cot 38− = 449.7055...m M 450 m 307 m • cot 38− x = cot 27− D cot 38− = 575.5968...m M 576 m 50 5. The answers are quite close. θ 6. The actual height is 1454 ft, or 443.2 m. 10 20 7. The answers will be different. 8. The answers will vary. Chapter 3 • Application of trigonometric and circular functions Exploration 3-1b 1. Survey 3-1a X Y1 0 1. Use December temperatures for month 0. y (°F) 100 50 12 18 x (months) 24 Y1 180 0 10 .17 270 D1 20 .34 360 0 30 .5 450 1 40 .64 540 0 50 .77 630 D1 60 .87 720 0 70 .94 80 6 X 0 .98 90 1 y 1 12- 1 2. θ-dilation 360− = 30 ; y = cos 30θ y 90° 180° 270° 360° 450° 540° 630° θ 720° θ 12 1 1 12−3. In problem 1, θ-dilation is 360− = 30 . Here is t-dilation (if t represents time in months) 12 months 1 = 30 months / degree, so y = cos 30t 360− 4. θ-translation C7−; y H 30 cos (θ D 7) y θ 12 1 5. y H 78.3 C cos 30(θ D 7−) 238 / Precalculus survey solution with trigonometry: Resource Book Instructor, Volume 1 ©2003 Key Curriculum Press 3. Horizontal dilation: 16 Period: 60− Amplitude: 5 Phase displacement: 10− Vertical displacement: D3 y = D3 + 5 cos 6(θ D 10−) 2. X Y1 x Y1 180 D1 10 .98 270 0 20 .94 360 1 30 .87 450 0 40 .4 077 540 D1 50 .64 630 0 60 .5 720 1 70 .34 80 .17 0 1 90 4. The chart should agree. 5. y H D7.3301270189222 6. The answers will be different. Survey 3-2b 1. 0 y 1 90° 180° 270° 360° 450° 540° 630° θ 720° 50° 3 100° 150° 1 3rd sin 45− = 0.7071...; cos 65− = .4226...; point (45−, 0.71) is in the first chart and the point (65−, 0.42) is on the second. 2. y = 55 + 45 cos 12(θ + 3−) 4. sinD1 0.4 = 23.5781... −; 3. y = 55 + 45 sins 12(θ D 19.5−) cosD1 0.8 = 36.8698... −; correspond to the points (24−, 0.4) in the first chart and (37−, 0.8) on the second. 4. The graphs match. 5. y = 0.5 + 3.5 sin 0.9(θ D 500−) 5. D1 ≤ y ≤ 1 6. y = D60 + 40 sins 7.5(θ D 36−) 6. The answers will be different. There are many examples in the text. 7. 7. The answers will be different. y Increase, concaving down 100 Survey 3-2a 1. Horizontal dilation: 12 Period: 180− Amplitude: 3 Phase displacement: C70− Vertical displacement: C4 Inflection point 10 θ 12° y 8. The answers will be different. 4 θ 100° 200° Survey 3-3a 300° 1. y 1 2. Horizontal dilation: 30 Period: 12− Amplitude: 4 Phase displacement: D1− Vertical displacement: D2 θ 90° 180° 270° 360° 450° 540° y 4 θ 50° θ. tan θ = sin θ cos θ θ 3. Asymptotes are on θ = 90− + 180−n, where cos θ = 0. 4. Passes are on θ = 0− + 180−n, where sin θ = 0. Precalculus with trigonometry: Resource Book Instructor, Volume 1 ©2003 Key Curriculum Press Survey Solutions / 239 11. sec θ goes from concaving to concaving down (and vice versa) rather than through the points that are in the chart. 5. tan 45− = 1 Chart for 3, 4 and 5. y 12. The answers will be different. Survey 3-3b 1 θ 90° 180° 270° 360° 450° 540° 1. Horizontal dilation: 15 Period: 36− Horizontal translation: C7− Vertical dilation: 12 Vertical translation: C3 2. y 6. y 1 θ θ θ 90° 180° 270° 360° 450° 10° 540° 3. Horizontal dilation: 16 Period: 30− Horizontal translation: D9− Vertical dilation: 2 Vertical translation: D1 7. The charts should be the same. 8. 4. y = D1 + 2 cot 6(x + 9−) 1 θ 90° 180° 270° 360° 450° 540° 5. Horizontal dilation: 14 Period: 90− Horizontal translation: D10− Vertical dilation: 3 Vertical translation: C1 6. y 9. y 1 θ 10° 1 θ 90° 180° 270° 360° 450° 540° 10. The inflection points are at θ = 0− + 180−n. tan θ has no inflection points because it is constantly decreasing, except when it changes from low values back to high, not over the points that are in the graph. 7. Horizontal dilation: 12 Period: 180− Horizontal translation: C25− Vertical dilation: 3 Vertical translation: C4 8. 4 + 3 with 2(θ D 25−) 9. The answers will be different. 240 / Precalculus Survey Solution with Trigonometry: Resource Book Instructor, Volume 1 ©2003 Key Curriculum Press Survey 3-4a 5. y 1. See drawing in problem 2. 1 2. Student drawings x π 1 2π 3π 4π 3. 1 radian = 57.2957... − 4. 1 radian = 360- 180- = = 57.2957... − 2π π 5. 3 radians = 3 • 6. y 1 180− = 171.8873... − π 1 x π 2π 3π 4π 6. The answers will be different. Survey 3-4b 7. Time for sinus and cosine H 360− H 2π radians; for tangential H 180− H π radians. 1.–4. Student drawings. 8. y = 3 + 2 cos 5. 1 radian = 57.2957... − 2π (x D 1) 10 9. The answers will be different. 6. 2π radians 7. 1 radian = 360− 180− = = 57.2957... − 2π π survey 3-6a 1. x H D4.5, D0.5, 8.5, 12.5, 21.5, 25.5 180− 8. 3 radians = 3 • = 171.8873... − π 2. y = 9 + 7cos 9. The ratio of arc length to radius is the same for a circle of any size. 3. 2π (x D 4) 13 y 10. The answers will be different. 16 Survey 3-5a 1. y x 1 θ 720° 360° 1 10 20 Graphs agree. 4. Yes 2. 5. x H D4.492 y 1 θ 720° 360° 1 6. x H D0.508, 8.508 7. x H 12.492 8. n H 2: x H D4.492 C 26 H 21.508 n H 1: x H 12.492 C 13 H 25.492 3. y 9. n = 1 76: x = 1000.4915... 1 θ 720° 360° 1 Sinusoid rises because a multiple of the period has been added to 12.4915, where the sinusoid goes up. 10. Responses will vary. 4. y 1 x π 2π 3π 4π Precalculus with trigonometry: Instructor Resource Book, Volume 1 ©2003 Key Curriculum Press Survey Solutions / 241 21 R D 500 + 2πn cosD1 2π 300 31.1889 ... days ≤ x ≤ 36.8110... days January 31 ≤ x ≤ 5 February Survey 3-6b 6. x = 13 ± 1. y = 12.9764... 2. 9 + 7 cos 2π (x D 4) = 5 13 7. The answers will be different. 2π 7 cos (x D 4) = D4 13 4 2π (x D 4) = D cos 13 7 4 2π (x D 4) = arccos D 13 7 13π 4 xD4= arccos D 2 7 13π 4 x=4+ arccos D 2 7 4 13 ±cosD 1 D + 2πn x=4+ 2π 7 13 4 D1 x=4+ cos D + 13n or 2π 7 13 4 4D cosD1 D + 13n 2π 7 Survey 3-7b 1. y = 2000 + 500 cos π (x + 30) 70 2. y x 200 400 x1 x2 D4.491... 0 8.508... D0.508... 1 21.508... 12.491... 2 34.508... 25.491... 600 800 3. 795.6617... ≤ x ≤ 824.3382... y x 200 n 800 1000 3. D1 600 400 1000 4. x = D4.491..., D0.508..., 8.508..., 12.491..., 21.508..., 25.491... N-values are listed in issue 4.4. D2000 + 500 cos 500 cos 5. x = 1308.508..., 1299.491... cos 6. x = 1000.491...; n = 77 π (x + 30) = D1600 70 π (x + 30) = 400 70 π (x D 30) = 0.8 70 π (x + 30) = ±0.6435... + 0.6435... + 0.602πn 70 x + 30 = ± 14.3382... + 140n x = D15.6617... + 140n or D44.3382... + 140n n = 6: x = 5 840 D 15.6617... or 840 D 44.3382... x = 795.6617... or 824.3382 7. The answers will be different. Survey 3-7a 1. y (number of red blood cells) 5. This would make a difference because the period would be 2 • 68, or 136, instead of 140. 1000 6. The answers will be different. x (days) 10 2. R = 500 + 300 cos 20 30 40 Survey 3-8a 50 2π (x D 13) 21 1-3. in 3 3. The graph corresponds to the sketch from part a. 4. 19 March H day 78 (or 79 in leap year) R(78) = 747.8716... R(79) = 687.0469... The patient will feel good on his birthday, if it is not a leap year, so-so otherwise. 5. The chart is over 700 on March 19 for a non-leap year, lower than otherwise. x 2 1 1 u 1 1 242 / Precalculus survey solution with trigonometry: Resource Book Instructor, Volume 1 ©2003 Key Curriculum Press 4. 3 radians = 3 • 5. 240− = 240− • 4. cos2 0.6 + sin2 0.6 = 1,180− = 171.8873... π in 4π π = radians 180− 3 sins 0.6 1 7 6. radians 10 u cos 0.6 7. y 5.5. Pythagorean property 3 6. cos2 x + sin2 x = 1 1 1 • (cos2 x + sin2 x) = 2 cos2 x cos2 x sin2 x 1 + = cos2 x cos2 x sin x 2 1 2 = 1+ cos x cos x 1 + tan2 x = sec2 x x 5 8. y = 5 D 2 cos 10 15 20 25 π π x or y = 5 + 2 cos (x D 4) 4 4 9. y(27) = 6,4142...m M 6.4 m 10. x = 14,9872...m M 15.0 m 11. 7 feet π (x D 1) = 7 ⇒ x = 1 a.m. 5.8 Period = 2 • 5.8 = 11.6 hours 3 + 4 cos π (x D 1) = D1 ⇒ x = 6.8 = 56:48 a.m. 5.8 1 foot deep 12. 3 + 4 cos 13. 16:00 p.m. is x = 16; y(16) = 1.9298... ft M 1.9 ft, which mat